
ADDITIONAL MATHEMATICS

4037/22

Paper 2

October/November 2016

MARK SCHEME

Maximum Mark: 80

Published

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Page 2	Mark Scheme	Syllabus	Paper
	Cambridge O Level – October/November 2016	4037	22

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

Question	Answer	Marks	Part Marks
1	$4x - 3 = x \rightarrow x = 1$ $4x - 3 = -x$ $x = 0.6$ OR $(4x - 3)^2 = x^2$ $15x^2 - 24x + 9 = 0$ $3(x - 1)(5x - 3) = 0$ $x = 1$ and $x = 0.6$	B1 M1 A1 B1 M1 A1	www use of $-x$ or $-(4x - 3)$ but not both. solve correct 3 term quadratic www
2	$a(\sqrt{3} - 1) + b(\sqrt{3} + 1)$ $= (\sqrt{3} - 3)(\sqrt{3} - 1)(\sqrt{3} + 1)$ $= 2(\sqrt{3} - 3)$ oe $a + b = 2$ $-a + b = -6$ $b = -2$ and $a = 4$	M1 DM1 A1 DM1 A1	Common denominator or $\times (\sqrt{3} - 1)(\sqrt{3} + 1)$ equate constant terms and $\sqrt{3}$ terms. both correct solve two linear equations to obtain $a =$ or $b =$ both correct
3	$2\lg x = \lg x^2$ $1 = \lg 10$ $\lg x^2 - \lg\left(\frac{x+10}{2}\right) = \lg\left(\frac{2x^2}{x+10}\right)$ oe $2x^2 - 10x - 100 = 0 \rightarrow 2(x+5)(x-10) = 0$ $x = 10$ only	B1 B1 B1 M1 A1	soi anywhere soi anywhere soi division; logs may be removed obtain correct 3 term quadratic equation and attempt to solve $x = -5$ must not remain.

Page 3	Mark Scheme	Syllabus	Paper
	Cambridge O Level – October/November 2016	4037	22

Question	Answer	Marks	Part Marks
4 (i)	$t = 10 \rightarrow N = 7000 + 2000e^{-0.5}$ $= 8213$ or 8210	B1	Do not accept non integer responses.
(ii)	$N = 7500 \rightarrow 7500 = 7000 + 2000e^{-0.05t}$ $e^{-0.05t} = \frac{500}{2000}$ $-0.05t = \ln 0.25 \rightarrow t = \frac{\ln 0.25}{-0.05}$ $= 27.7$ (days)	M1 M1 A1	insert and make $e^{-0.05t}$ subject take logs and make t the subject awrt 27.7
(iii)	$\frac{dN}{dt} = -100e^{-0.05t}$ $t = 8 \rightarrow \frac{dN}{dt} = \pm 67$ (.0)	M1 A1 A1	$ke^{-0.05t}$ where k is a constant $k = -100$ or -0.05×2000 awrt ± 67 mark final answer
5 (i)	$\frac{dy}{dx} = 3x^2 + 4x - 7$ $x = -2 \rightarrow \frac{dy}{dx} = 12 - 8 - 7 = -3$ Equation of tangent : $\frac{y-16}{x+2} = -3 \rightarrow y = -3x + 10$	B1 M1 A1	insert $x = -2$ into <i>their</i> gradient and use $(-2, 16)$ and <i>their</i> gradient of tangent in equation of line.
(ii)	Tangent cuts curve again $x^3 + 2x^2 - 7x + 2 = -3x + 10$ $x^3 + 2x^2 - 4x - 8 = 0$ $(x+2)(x+2)(x-2) = 0$ $x = 2, y = 4$	M1 A1 M1 A1A1	equate curve and <i>their</i> linear answer from (i). factorise: $(x \pm 2)$ and a two or three term quadratic is sufficient. Allow long division withhold final A1 if $(2, 4)$ not clearly identified as their sole answer.
6 (i)	$\frac{\cos x}{1 + \tan x} - \frac{\sin x}{1 + \cot x} = \frac{\cos x}{1 + \frac{\sin x}{\cos x}} - \frac{\sin x}{1 + \frac{\cos x}{\sin x}}$ $= \frac{\cos^2 x}{\cos x + \sin x} - \frac{\sin^2 x}{\cos x + \sin x}$ $= \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\cos x + \sin x)}$	M1 M1 A1 A1	$\tan x = \frac{\sin x}{\cos x}$ and $\cot x = \frac{\cos x}{\sin x}$ Attempt to multiply by $\cos x$ and $\sin x$ AG
(ii)	$-\sin x + \cos x = 3\sin x - 4\cos x$ $5\cos x = 4\sin x$ $\tan x = \frac{5}{4}$ $x = 51.3^\circ, -128.7^\circ$	M1 A1 A1A1	equate and collect $\sin x$ and $\cos x$ oe FT from $\tan x = k$

Page 4	Mark Scheme	Syllabus	Paper
	Cambridge O Level – October/November 2016	4037	22

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7 (i)	$h = \sqrt{9 - x^2}$ $A = \frac{\sqrt{9 - x^2}}{2}(14 + x + x) = \sqrt{9 - x^2}(7 + x)$	B2/1/0	Must be clear that $\sqrt{9 - x^2}$ is the height of the trapezium. $14 + 2x$ oe must be seen AG
(ii)	$\frac{dA}{dx} = \sqrt{9 - x^2} + (7 + x) \frac{1}{2}(9 - x^2)^{-0.5} \times -2x$ $\frac{dA}{dx} = 0 \rightarrow 9 - x^2 = 7x + x^2$ $2x^2 + 7x - 9 = 0$ $x = 1$ $A = 16\sqrt{2} \text{ or } 8\sqrt{8} \text{ or } \sqrt{512} \text{ or } 22.6$	M1 A2/1/0 M1 A1 A1 A1	product rule on correct function minus 1 each error, allow unsimplified. equate to 0 and simplify to a linear or quadratic equation. correct three term quadratic obtained Extra positive answer loses penultimate A1 . ignore negative solution.
8 (i)	$f'(x) = \frac{(x^3 + 1)9x^2 - (3x^3 - 1)3x^2}{(x^3 + 1)^2}$ $= \frac{12x^2}{(x^3 + 1)^2}$	M1 A1 A1	quotient rule or product rule all correct www beware $9x^6 - 9x^6$ gets A0
(ii)	$\int_1^2 \frac{x^2}{(x^3 + 1)^2} dx = \frac{1}{12} \left[\frac{3x^3 - 1}{x^3 + 1} \right]_1^2$ $= \frac{1}{12} \left[\frac{23}{9} - \frac{2}{2} \right]$ $= \frac{7}{54}$	M1 A1 DM1 A1	$c \times \frac{3x^3 - 1}{x^3 + 1}$ FT $c = \frac{1}{\text{their } 12}$ top limit – bottom limit in <i>their</i> integral. or 0.130 or 0.1296 or 0.12
(iii)	$x = \frac{3y^3 - 1}{y^3 + 1}$ $y^3 = \frac{x + 1}{3 - x}$ $f^{-1}(x) = \sqrt[3]{\frac{x + 1}{3 - x}}$ $\text{Domain : } -1 \leq x \leq 2\frac{6}{7}$	B1 B1 B1 B1 B1	make y^3 or x^3 the subject FT take cube root (as long as y^3 or x^3 equals a fraction with terms in x or y only) oe FT change x and y – can be done at any time Allow upper limit of 2.86. Do not isw

Page 5	Mark Scheme	Syllabus	Paper
	Cambridge O Level – October/November 2016	4037	22

Question	Answer	Marks	Part Marks
9 (i)	tangent touches circle $x^2 + (kx - 4)^2 - 2(kx - 4) = 8$	M1	eliminate y or x allow unsimplified
	$k^2x^2 + x^2 - 8kx - 2kx + 16 = 0$ or better	A1	
	Equal roots as tangent touches circle : $b^2 = 4ac$ $(-10k)^2 = 4(k^2 + 1) \times 16$ $36k^2 = 64$ $k = +\frac{4}{3}$ only	DM1 A1 A1	use of discriminant on 3 term quadratic soi oe any inequality loses last A1
(ii)	$x = \frac{-b}{2a} \rightarrow x = \frac{\frac{4}{3} \times 10}{\frac{25}{9}}$	M1	use $x = \frac{-b}{2a}$
	$x = \frac{12}{5} \quad y = -\frac{4}{5}$	A1A1	
	OR tangent $y = \frac{4}{3}x - 4$ cuts radius	M1	find equation of radius and attempt to solve with tangent
	$y = -\frac{3}{4}x + 1$		
	at $x = \frac{12}{5}$	A1	
	$y = -\frac{4}{5}$	A1	
(iii)	OR Obtain $25x^2 - 120x + 144 = 0$ oe	M1	obtain any 3 term quadratic using <i>their</i> non zero k and reach $x = \dots$
	$(5x - 12)(5x - 12) = 0$		
	$x = \frac{12}{5} \rightarrow y = -\frac{4}{5}$	A1A1	
	$TP = \sqrt{(0 - 2.4)^2 + (-4 + 0.8)^2} = 4$	M1A1	M1 for using <i>their</i> T and $(0, -4)$. Signs must be correct.

Page 6	Mark Scheme	Syllabus	Paper
	Cambridge O Level – October/November 2016	4037	22

Question	Answer	Marks	Part Marks
10 (i)	$r_j = \begin{pmatrix} 5000 \\ 1000p \end{pmatrix} + \begin{pmatrix} -2\cos 40 \\ 2\cos 50 \end{pmatrix} t$	B1 B1	x coordinate oe y coordinate oe
(ii)	$2.5t\cos 70 = 5000 - 2t\cos 40$ $t = \frac{5000}{2.5\cos 70 + 2\cos 40}$ $= 2095$ awrt or 2090 or 2100 $(2.5\cos 20 - 2\cos 50) \times 2095 = 1000p$ $p = 2.23$ awrt	M1 DM1 A1 M1 A1	equate <i>their</i> x values (must be 3 terms) make <i>t</i> the subject allow one sign error equate <i>their</i> y values (must be 3 terms) and insert <i>their</i> <i>t</i> or $ t $.
11 (i)	Free choice : no. of ways ${}^6C_4 \times {}^5C_2 = 15 \times 10$ $= 150$	B1 B1	${}^6C_4 \times$ another nC_r term only $\times {}^5C_2$ and answer or vice versa
(ii)	Both Mr and Mrs Coldicott ${}^5C_3 \times {}^4C_1 = 10 \times 4$ $= 40$	B1 B1	${}^5C_3 \times$ another nC_r term only $\times {}^4C_1$ and answer or vice versa
(iii)	Mr C and not Mrs C ${}^5C_3 \times {}^4C_2 (= 60)$ Not Mr C and Mrs C ${}^5C_4 \times {}^4C_1 (= 20)$ Total = 80 OR Total = (i) – (ii) – neither Neither = ${}^5C_4 \times {}^4C_2 = 30$ Total = $150 - 40 - 30 = 80$	B1 B1 B1 M1 A1 A1	An incorrect final answer does not affect the awarding of the first two B1 marks. www